St Hilda's
ANGLICAN SCHOOL FOR GIRLS

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| **Time:** | 20 minutes |
| **Total Marks:** | 19 marks |

# Year 12 Methods

# Review Response 2

Wednesday Sept 8th 2021

**Resource Free**

**ClassPad calculators are Not permitted.**
**Formulae Sheet is Permitted.**

**Name:** Solutions

1. (2, 2 = 4 marks)

Find $\frac{dy}{dx}$ for each of the following. Do not simplify.

a) $y = \log_e\left(x + x^2 + x^3\right)$

$$\frac{dy}{dx} = \frac{1 + 2x + 3x^2}{x + x^2 + x^3}$$ ✓✓

b) $y = x^2 \ln(2x)$

✓ ✓

$$\frac{dy}{dx} = (2x)\ln(2x) + \frac{2}{2x}\left(x^2\right)$$

2. (1 marks)

Solve $3^x = 5$ giving the exact answer using base ten logarithms.

$$\log 3^x = \log 5$$
$$x \log 3 = \log 5$$
$$x = \frac{\log 5}{\log 3}$$

3. (3 marks)

Find an expression (which does not contain log functions) for $p$ in the following.

$3 - 2\log_3 y = \log_3 p$

$$\log_3 p + \log_3 y^2 = 3$$
$$\log_3 \left(py^2\right) = 3$$
$$py^2 = 3^3$$
$$p = \frac{27}{y^2}$$

4. (2 marks)

A continuous random variable $X$ is uniformly distributed on the interval $-2 \leq X \leq 4$.

Determine $\mathrm{P}(X \leq 2 \mid X \geq 1)$.

$$P(x \leqslant 2 \mid x \geqslant 1) = \frac{P(1 \leqslant x \leqslant 2)}{P(x \geqslant 1)}$$
$$= \frac{1 \times \frac{1}{6}}{3 \times \frac{1}{6}}$$
$$= \frac{1}{3}$$

5. (3 marks)

Differentiate the following. Simplify your answers.

$$y = \ln\left(\frac{cos(x)}{e^{2x}}\right)$$

$$y = \ln(\cos x) - \ln e^2$$
$$= \ln(\cos x) - 2x$$
$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} - 2$$
$$= -\tan x - 2$$

$$\frac{-\sin x + 2\cos x}{\cos x}$$

6. (2 marks)

Evaluate $\log(100) - \ln\left(e^{-3}\right)$.

$$= 2 + 3$$
$$= 5$$

7. (2 marks)

If $\log_3 a = p$ and $\log_3 b = q$, express the following in terms of $p$ and/or $q$.

$$\log_3\left(\frac{a^2 b^3}{9}\right)$$

$$= \log_3 a^2 b^3 - \log_3 9$$
$$= \log_3 a^2 + \log_3 b^3 - 2$$
$$= 2\log_3 a + 3\log_3 b - 2$$
$$= 2p + 3q - 2$$

8. (2 marks)

If $P = 5e^{(t-1)}$ find an exact expression for $t$ in terms of $P$.

$$\ln \frac{P}{5} = \ln e^{(t-1)}$$

$$\ln \frac{P}{5} = t - 1$$

$$t = \ln\left(\frac{p}{5}\right) + 1$$

or

$$t = \ln P - \ln 5 + 1$$

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| **Time:** | 25 minutes |
| **Marks:** | 21 marks |

# Year 12 Methods

# Review Response 2

Wednesday Sept 8th 2021

**Resource Assumed**

**ClassPad calculators are permitted.**

**Formulae Sheet is Permitted.**

**Name:**

9. (3 marks)

Through geological surveys and test drilling, a company discovers a new oil field off the coast Cottesloe. The experts predict that profitable extraction of the oil can be carried out and that in any one year this extraction will reduce the quantity of oil remaining in the field by 10% of what it was at the beginning of that year. Extraction will become unprofitable when just 45% of the original quantity remains. For how many years can the company expect the field to remain profitable?

$$A = A_o \cdot e^{kt}$$ ✓

$$0.9A_o = A_o \cdot e^{k(1)}$$

$$k = \ln(0.9)$$ ✓

$$0.45A_o = A_o \cdot e^{\ln(0.9)t}$$

$$t = 7.58 \text{ years}$$ ✓

10. (2, 1, 1 = 4 marks)

A random variable $X$ has probability density function:

$$f(x) = \begin{cases} ae^{-3x} & \text{for} \quad x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

a) Solve for $a$

$$\int_0^\infty ae^{-3x} dx = 1$$

$$a = 3$$

b) Find the mean of $X$

$$\mu = \int_0^\infty x\left(3e^{-3x}\right) dx$$

$$= \frac{1}{3}$$

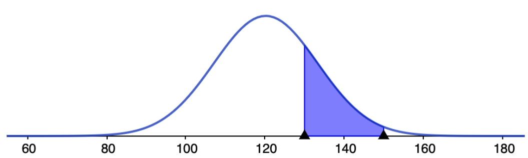
c) Find the variance of $X$

$$\sigma^2 = \int_0^\infty \left(x - \frac{1}{3}\right)^2 \left(3e^{-3x}\right) dx$$

$$= \frac{1}{9}$$

11. (2 marks)

Let us suppose that the monthly rainfall in a region is normally distributed with a mean of 120.2 mm and a standard deviation of 13.1 mm. In a year, how many months would you expect this region to have a rainfall that is a between 130 mm and 150 mm?




$$N \sim \left(120.2,\ 13.1^2\right)$$

$$P(130 \le x \le 150) = 0.2157$$

$0.2157 \times 12 = 2.59$ months

12. (2, 3 = 5 marks) (+1 units)

A particle moves in a straight line such that its displacement from a fixed point $\mathrm{O}$, at time $t$ seconds $(t \geq 0)$, is $x$ metres where $x = 8\ln(1+t) - 3t$.

a) Find $t$ when the velocity is zero.

$$\begin{aligned} V(t) &= \frac{dx}{dt} \\ &= \frac{8}{1+t} - 3 \text{ m/s} \\ 0 &= \frac{8}{1+t} - 3 \\ &= \frac{5}{3} \text{ s} \end{aligned}$$

b) Find $t$ when the velocity (m/s) is numerically equal to the acceleration (in $\mathrm{m/s^2}$).

$$\begin{aligned} a(t) &= \frac{dv}{dt} \\ &= \frac{-8}{(1+t)^2} \\ a(t) &= v(t) \\ t &= \frac{1}{3} - \frac{2\sqrt{10}}{3} \text{ or } \frac{1}{3} + \frac{2\sqrt{10}}{3} \\ &= 2.44 \text{ s} \end{aligned}$$

Must show both solutions.

13. (2, 1 = 3 marks)

The discrete random variable $X$ is binomially distributed with $X \sim \mathrm{Bin}(n, p)$. If $\mathrm{E}(X) = 50$ and $\mathrm{SD}(X) = 5$ find $n, p$ and $\mathrm{P}(X \leq 50)$ giving the last of these correct to 4 decimal places.

$$\begin{aligned} 50 &= np \\ np(1-p) &= 5^2 \\ \therefore n = 100,\ p &= \frac{1}{2} \\ P(x \leqslant 50) &= 0.5398 \end{aligned}$$

14. (2, 1 = 3 marks)

232 fish from a local lake were captured, tagged and released back into the lake. 150 fish were captured in a second sample and 35 were found to have a tag.

a) Find an approximation for the total number of fish in the lake.

$$\frac{N}{232} = \frac{150}{35}$$
$$N = 994.285$$
$$N \approx 1000 \text{ fishys}$$

b) Give an assumption that needed to be made for the above capture-recapture process to produce a valid estimation for the population of fish in the lake.

Tagged fish disperse randomly throughout the population.

etc.